

Rubber Bandzzles: Three Mathematical Puzzle-Art Challenges

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Abstract It is possible to ignore many boring office meetings if you can distract yourself with handy office supplies. Should there be rubber bands lying about on the conference table of your next dreary meeting, here are three artistic challenges for entertaining yourself: *The Worm*, *Infinity Squared*, and *Pentadigitation*.

1. *The Worm*

The Worm is a miniature sculpture and a study in conservation of twist. I've enjoyed replicating it hundreds of times over the years. As Fig. 1 indicates, *The Worm* is a single rubber band very tightly wound. It is twisted as in a rubber-band powered propeller airplane ready to go, but completely stable. It does not untwist even if picked up and stretched. Can you discover how to make one before reading my solution below?



Fig. 1: *The Worm*.

2. *Infinity Squared*

Infinity Squared is a kinetic sculpture which you make with one rubber band and the thumbs and two fingers of each hand. To warm up to it, first make the very simple motion I call *Zero*, shown in Fig. 2a, which is basically twiddling your thumbs with a rubber band not slipping around them. Then make “Infinity” which uses the thumb and forefinger of each hand inside a rubber band that has been twisted into a figure-eight shape. Since the crossed loop naturally sits horizontally and you can do the motion forever, I think of it as ∞ . In a sense, your thumbs together make one pulley and your index fingers make another, and the rubber band is a crossed conveyor belt. You drive it by bringing the thumb and forefinger of one hand together through the other pair, then switching which hand is on the inside. (Try it in reverse as well.) After mastering *Infinity*, you will be ready for the *Infinity Squared* challenge, which is like *Infinity*, but with three loops and two crossings. You alternate between the positions shown in Figs. 2c and 2d. The six-finger motion that keeps it going smoothly is addictive once you master it. (Be sure to try it in the reverse direction as well.)

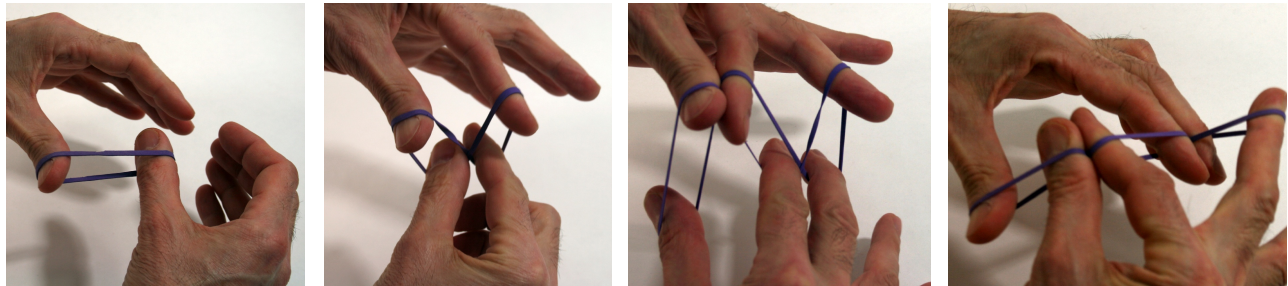


Fig. 2: (a) *Zero*, (b) *Infinity*, and (c, d) two positions in the cyclic motion of *Infinity Squared*.

3. *Pentadigitation*

The goal of this mathematical performance art is to amaze or amuse your friends and coworkers by making a pentagram around the five fingers of one hand, using just the fingers of that hand. Start with a rubber band loose in the palm, and with just that hand, manipulate it to match Fig. 3. Once you learn to make one, it is surprisingly easy (via neuronal symmetry?) to do one star simultaneously in each hand.

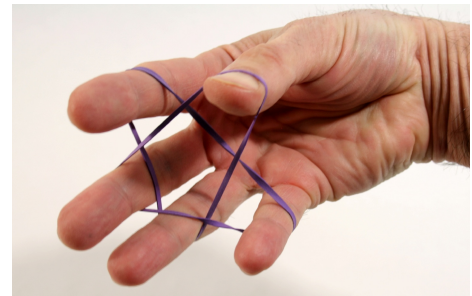


Fig. 3: Rubber band pentagram

SOLUTIONS

1. *The Worm* The secret is that one half is twisted clockwise and the other half is twisted counterclockwise, with a tight little knot at the juncture to prevent the halves from canceling. To make it, repeatedly roll the rubber band along the thumbs and forefingers, continuing as many times as possible, and ending up as in Fig 4a. One side is wound tightly clockwise, the other is wound tightly counterclockwise, and finger pressure keeps it from unwinding. Then bring the ends together in one hand and hold the center as in Fig. 4b to keep it from unwinding. The other hand is now available to make a simple overhand knot at the center. Tighten the knot to be as small as possible and then temporarily untwist, stretch, and redistribute the turns in each half to be straight, neat, and even as in Fig. 1.

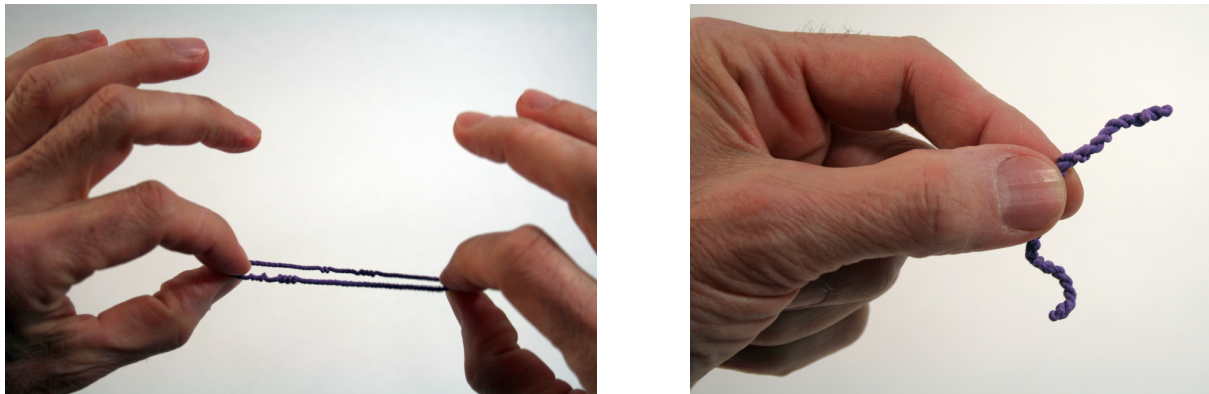


Fig. 4: Steps in Making *The Worm*.

2. *Infinity Squared* I know of no secret to this other than practice. The fingers will smoothly alternate between the two positions shown in Figs. 2c and 2d as the rubber band moves along without slipping. I've been doing this since I was a kid, so it seems like a natural motion to me, but your mileage may vary. (After you master it, for extra credit try *Infinity Cubed* and so on, until you run out of fingers.)

3. *Pentadigitation Puzzle* This can be solved several ways, depending on how flexible your thumb and fingers are. I like to first get the rubber band on fingers 4 and 5 (ring finger and pinkie) as in Fig. 5a. Dip the thumb into the loop and pull the far side across the palm to get to the position of Fig. 5b. Dip finger 3 into the thumb-loop, pull up the side furthest from that finger, then drop the loop from the thumb, to get to the position of Fig. 5c. Lift the near part of the finger-5 loop with the thumb, and then let finger 2 grab it from below, and release the thumb, to get to the position of Fig. 5d. Finally, the thumb can lift the short center band marked *X* to obtain the pentagram of Fig. 3.

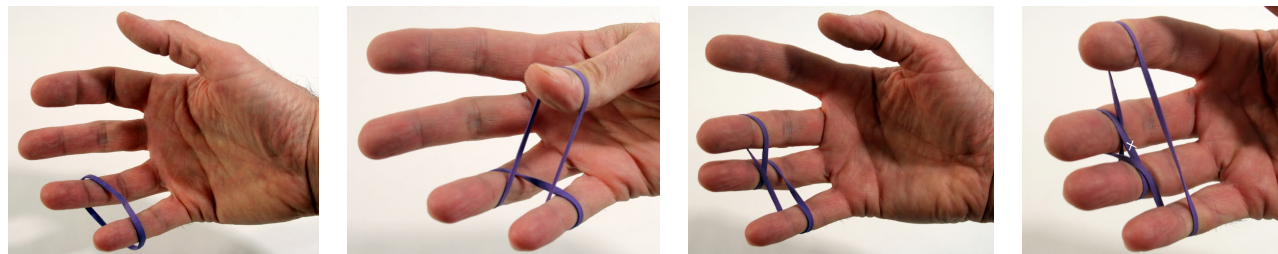


Fig. 5: Steps in making *Pentadigitation*.

Conclusions

Mathematical rubber-band puzzle-sculpture, kinetic art, and performance art are possible on a small scale. I would be pleased if copies of these little artworks are soon enlivening boring conference rooms all around the world.